Lesson plan: What's the Equation?

Learning objective

Students will construct linear and exponential functions from given graphs, descriptions, or input-output pairs. They will also relate the domain of these functions to their graphs and the quantitative relationships they describe.

Student-facing objective: By the end of this lesson, I can create linear and exponential functions from graphs or data. I can explain how the domain relates to the graph and the situation it represents.

Standards:

- HSF.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- HSF.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Learning activities

Warm-up

Which one doesn't belong?

Display four graphs of different functions:

- Graph A: Linear function $y = 2x + 1$
- Graph B: Exponential function $y = 2^x$
- Graph C: Quadratic function $y = x^2$
- Graph D: Linear function $y = -x + 3$

Ask students to decide which graph doesn't belong and explain their reasoning. Encourage multiple perspectives and justifications. Record their explanations for all to see.

Direct instruction

1. **Introduce Linear Functions:**
    - Display a graph of a linear function, e.g., $y = 3x + 2$.
    - Explain the components: slope and y-intercept.
    - Discuss the domain of the function and its real-world context, e.g., cost of items where x is the number of items and y is the total cost.
    - Provide a step-by-step example of constructing the equation from the graph. **[Provide solution steps]**

2. **Introduce Exponential Functions:**
    - Display a graph of an exponential function, e.g., $y = 2^x$.
    - Explain the components: base and exponent.
    - Discuss the domain of the function and its real-world context, e.g., population growth where x is time in years and y is the population.
    - Provide a step-by-step example of constructing the equation from the graph. **[Provide solution steps]**

3. **Relate Domain to Graphs:**

    - Present a scenario: "A factory produces n engines, and the function $h(n) = 5n$ gives the number of person-hours required."
    - Discuss why the domain is positive integers.
    - Show how to identify the domain from the graph and relate it to the context.
    - Provide a step-by-step example of determining the domain from a given graph and context. **[Provide solution steps]**

Guided practice

Think, pair, share

1. **Think**: Present students with a graph of a linear function, e.g., $y = 4x - 3$, and an exponential function, e.g., $y = 3^x$. Ask them to individually write down the equations of these functions and identify the domain for each.
2. **Pair**: Have students pair up and compare their equations and domain identifications. Encourage them to discuss any differences and come to a consensus.
3. **Share**: Select pairs to share their findings with the class. Record the equations and domains on the board.
4. **Class Discussion**: Facilitate a discussion on the different approaches used to determine the equations and domains. Highlight correct methods and clarify any misconceptions.
5. **Reinforcement**: Provide another set of graphs (one linear, one exponential) and repeat the process to reinforce understanding.

Guided/Independent practice

There are 10-12 slides from illustrative math to work through

- Construct the equation of a linear function from a given graph.
- Construct the equation of an exponential function from a given graph.
- Identify the domain of a given function and relate it to its graph and context.

Circulate throughout the class to observe students as they work and provide support as needed.

Exit ticket

Ask students to answer these questions on their way out:

1. How do you determine the equation of a linear function from its graph?
2. What is the domain of the function $y = 2^x$ and why?

Here are some suggested answers:

1. Suggested answer to Question 1: Identify the slope and y-intercept from the graph and use the form $y = mx + b$.
2. Suggested answer to Question 2: The domain is all real numbers because the exponential function is defined for all real x.

Teacher resources

Lesson plan: What's the Equation?

Differentiation guide

- **Advanced learners:**
  - Challenge with more complex functions, such as piecewise or logarithmic.
  - Introduce real-world problems requiring multi-step solutions.
  - Encourage exploration of domain restrictions in non-standard contexts.

- **Striving learners:**

  - Provide step-by-step scaffolding for constructing equations.
  - Use visual aids and manipulatives to illustrate concepts.
  - Offer additional practice with simpler functions before progressing.

Notable definitions

- **Linear Function**: A function that creates a straight line when graphed. It has the form $y = mx + b$, where m is the slope and b is the y-intercept.
- **Exponential Function**: A function where the variable is in the exponent. It has the form $y = a \cdot b^x$, where a is a constant and b is the base.
- **Domain**: The set of all possible input values (usually x) for which the function is defined. For example, the domain of $y = 2^x$ is all real numbers.

Required materials

- Graph paper
- Colored pencils
- Calculators
- Whiteboard and markers
- Projector or smartboard
- Handouts with pre-drawn graphs
- Worksheets for independent practice
- Exit ticket slips

Lesson summary

- Warm-up (5 min)
- Direct instruction (15 min)
- Guided practice (20 min)
- Guided & Independent practice (30 min)
- Exit ticket (5 min)